Five Year Integrated M.Sc. 2022
Subject: Integral Equations and Calculus of Variations
Course Code: -MT-4-7-5
Full Marks.-40
Time: 3 Hrs.

Attempt any four questions.

1. (a) Convert the following differential equations into integral equation

$$y''(t) + 2y'(t) - 8y(t) = 5t^2 - 3t, \qquad y(0) = -2, y'(0) = 3$$

(b) Convert the following integral equations into differential equation and associated conditions

$$y(t) = 5\cos(t) + \int_0^t (t-u)y(u)du$$

5+5=10

2. (a) Solve the following integral equations:
(i)

$$y(x) = x^2 + \lambda \int_0^1 (1 - 3x\xi)y(\xi)d\xi,$$

(ii)

$$u(x) = \sin x + \lambda \int_0^{\pi} \sin x \sin(2y)u(y)dy, ,$$

for all values of λ. 5+5=10

3. (a) Prove that all the eigen values of a real symmetric kernel are real.

(b) Show that the equation

$$g(s) = \lambda + \int_0^{\pi} (\sin s \sin 2t)g(t)dt$$

only has the trivial solution.

5+5=10

4. State convolution theorem of functions. Hence solve

$$\int_0^t y(u)\cos(t-u)du = y'(t) \qquad y(0) = 1.$$

Show that the integral equation of the above integro -differential equation can be expressed as the integral equation

$$1 + \int_0^t (t-u)y(u)\cos(t-u)du = y(t).$$

5+5=10

5. (a) Reduce the following equation into an equation of the second kind

$$\int_0^x \cos(x-t)y(t)dt = x$$

(b) Determine values of the real numbers K, L for which the integral equation

$$y(x) = 1 + Kx + Lx^2 + \frac{1}{2}\int_{-1}^{1}(1+3xt)y(t)dt \qquad (x \in [-1,1])$$

has a solution, and find the solutions of this equation.

3+7=10

6. Find Resolvent Kernel of

$$y(x) = 1 + \int_0^x y(t)dt$$

hence solve it. 7+3=10